GCSE Maths – Number

Ratio Problems

Notes

WORKSHEET

Ratios

A ratio tells us the **proportions** of how things are shared. For example, a ratio of $1:1$ means that the objects are shared **equally**, whereas a ratio of $2:1$ means that twice as many objects are in the first group compared to the second group.

We need to know how to find a ratio. Typically, we are told how the objects are shared between two or more groups. We then write this as a ratio $x:y$.

Ratios can be **simplified** by dividing each number by the **highest common factor**.

Example: There are 9 apples in a bag. Liam gets 6 apples and Mia gets 3. Write out the simplified ratio for how the apples are shared.

We are told how many apples Liam and Mia each get.
To find the ratio, write out the numbers for each:

$$6:3$$

Simplify this ratio by dividing each number by the highest common factor:

$$2:1$$

The ratio for the number of apples for Liam to Mia is $\mathbf{2:1}$.

Ratios can also be treated as **fractions**. Each number of the ratio tells us the **proportion** of the **total** that belongs to each group. For example, using the question above, Liam gets $\frac{6}{9}$ of the total apples, whereas Mia gets $\frac{3}{9}$. The total amount is on the bottom of the fraction (The denominator).

Using Ratios to Generate Fractions

To work out a fraction from a ratio, we must first calculate the **total number of 'parts'** by adding together **each number** in the ratio. This number will be our **denominator**.
To find the numerator, we take the **appropriate part** from the ratio.

Example: Jill and Sam share sweets in a ratio of $3:4$. What fraction of the total sweets does Jill get?

First, calculate the denominator by finding the total number of parts.

$$3+4=7$$

There are 7 *parts in total so the denominator of the fraction will be* 7.

We are now looking for Jill's part of the total. Use the number given to Jill as the numerator.

Jill's fraction of sweets is $\frac{3}{7}$.

There can be **more than two groups** in the ratio (written as $x: y: z$ or more). If trying to find a fraction, we still follow the same method: find the **sum** of all the 'parts' for the denominator, then use the **specific number** for one of the groups as the numerator.

Example: There are red, blue and green marbles in a bag, in the ratio of $5 : 3 : 2$. What fraction of the marbles are blue?

First, find the denominator by adding together all the parts.

$$5 + 3 + 2 = 10$$

Now use the number for the blue proportion (3) as the numerator:

$$Fraction\ of\ marbles\ that\ are\ blue = \frac{3}{10}$$

Using Fractions to Generate Ratios

We may need to reverse the process and generate a ratio from a known fraction. For example, consider the following question:

- In a bag of counters, $\frac{3}{5}$ of all counters are blue. The rest are yellow. Write down the ratio for blue to yellow sweets.

For this type of question, we have been given the **sum** of the '**parts' of the ratio** already: it is the **denominator** of the fraction. We have also been given the **proportion** of one group: the **numerator**.

We can write down what we already know as:

- $Blue : yellow = 3 : x$
- $3 + x = 5.$

By writing it out like this, we can see that $x = 2$, which is the number of **'parts'** that are yellow. Therefore, the ratio of $Blue : Yellow = 3 : 2$.

Example: In a litter of dog pups, $\frac{1}{4}$ have brown fur and the rest have black fur. What is the ratio for brown fur to black fur?

We have been given the denominator, which is the sum of all the parts. We've also been given the number of parts that belong the first group (brown fur). We can write this as:

$$1 : x$$
$$1 + x = 4$$

We can see that $x = 3$, so the ratio of $Brown\ fur : Black\ fur = \mathbf{1 : 3}$.

The question may contain more than one fraction and ask us to find the ratio. If the denominators of each fraction are the same, then we can follow the same method – construct an equation to find the unknown parts of the ratio.

Example: At a cinema, $\frac{1}{10}$ of the films being shown are animated films, $\frac{4}{10}$ are comedies and the rest are adventure. Calculate the ratio for animated to comedy to adventure films.

Write out what we already know as:

$$1 : 4 : x$$

$$1 + 4 + x = 10$$

Using the equation above, we can calculate the unknown parts of the ratio:

$$5 + x = 10$$
$$x = 5$$

Therefore, the ratio is $Animated : Comedy : Adventure = \mathbf{1 : 4 : 5}$.

If the denominators of each fraction given are not the same, then we need to change the fractions to make the denominators the same before continuing.

Example: A café is selling soft drinks. $\frac{5}{12}$ of the drinks sold are Diet Coke, $\frac{1}{4}$ are Sprite and the rest are Fanta. What is the ratio for Diet Coke to Sprite to Fanta?

First, convert the known fractions to ensure they have the same denominator. Here, we can multiply the numerator and denominator of $\frac{1}{4}$ *by* 3 *to give us the same denominator as* $\frac{5}{12}$.

$$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

Now we can write out the parts of the ratio we already know and construct an equation to find the missing part.

$$5 : 3 : x$$
$$5 + 3 + x = 12$$

Solve for x*:*

$$8 + x = 12$$
$$x + 4$$

Now write out the full ratio.

$Diet\ Coke : Sprite : Fanta = \mathbf{5 : 3 : 4}$

Multiple Ratios

Sometimes, we may be asked to find the fraction of one group in a question that gives us **two ratios**. For example, consider the following question:

- At a swimming pool, the ratio of adults to children is $2:5$. The children are either boys or girls, and the ratio of boys to girls is $2:1$. What is the fraction of girls at the swimming pool?

We are dealing with multiple ratios here, which means we will be **converting the ratios** to **fractions** and **multiplying** them.

1. First, work out the **fraction of children** at the pool:
   - Take the **numerator** as the number of 'parts' that **represents children** (5).
   - **Add together** the total number of 'parts' to find the **denominator** (7).

   The fraction of children at the pool is $\frac{5}{7}$.
2. Next, we want to find the fraction of girls out of all children at the pool:
   - Take the numerator as the number of 'parts' that represents girls (1).
   - Find the denominator by taking the sum of the parts (3).

   The fraction of girls at the pool (in relation to all children) is $\frac{1}{3}$.
3. Now we **multiply** the two fractions together because we are looking for the fraction that represents those who are children AND girls.

   $$\frac{5}{7} \times \frac{1}{3} = \frac{5}{21}$$

   The fraction of girls at the pool is $\frac{5}{21}$.

We work out **each fraction in turn**, then **multiply** the **fractions** together to find the appropriate group.

Example: At a library, the ratio of fiction books to non-fiction books is $4:1$. Within the fiction books, there are science fiction, historical fiction and crime books in the ratio $6:4:3$. What fraction of the total books are science fiction?

First, work out the fraction of fiction books.

$$Fraction\ of\ fiction\ books = \frac{4}{4+1} = \frac{4}{5}$$

Next, calculate the fraction of science fiction books.

$$Fraction\ of\ science\ fiction = \frac{6}{6+4+3} = \frac{6}{13}$$

Multiply the two fractions together to find the fraction of total books that are science fiction:

$$\frac{4}{5} \times \frac{6}{13} = \frac{24}{65}$$

The answer is $\frac{24}{65}$.

Ratio Problems - Practice Questions

1. Naoki and Oscar share sweets from a bag in the ratio $4 : 8$. What fraction of the total sweets does Naoki get?

2. There are cows on a field. $\frac{1}{5}$ of the cows are red, and the rest are white. Calculate the ratio for red cows to white cows on the field.

3. A takeaway sells fish, chips and sausages. On one particular evening, $\frac{11}{32}$ of total sales are fish, $\frac{3}{8}$ are chips and the rest are sausages. Calculate the ratio for sales of fish to chips to sausages.

4. Shannon sells beige and white paint in a ratio $6 : 1$. The beige paint comes in either matte or glossy and is sold in the ratio $3 : 2$. What fraction of all paint sold is matte beige?

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.